

Second Korea-Russia Conference on Knot theory and Related Topics

The Second Russia-Korea Conference on Knot theory and Related topics will be held from 29th November to 3rd December in 2021 by Zoom.

Link for Zoom conference:

Join Zoom Conference: <https://us02web.zoom.us/j/85219461877>

Conference ID: 852 1946 1877

Passcode: 269588

The Conference Program of the KRCKT-2021 will include papers dealing with the following major themes:

- Invariants in knot theory
- Braid theory
- Virtual knot and braid theory
- Theory of G_n^k groups
- 2-knots and Higher dimensional knots
- Algebraic topology
- Geometric topology

Program committee

Yongju Bae	Sergei Gukov	Igor Nikonov
Louis Kauffman	Sang Youl Lee	Sergei Matveev
Andrei Vesnin	Vassily Manturov	

Organizing committee

Yongju Bae	Sang Youl Lee	Jieon Kim
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Hosting organizations

- Moscow Institute of Physics and Technology, Russia
- Pusan National University, Republic of Korea
- Kyungpook National University, Republic of Korea
- Tomsk State University, Russia
- Moscow State University, Russia

Timetable

Monday, 29th November

Korea time	Moscow time			
14:00–14:40	8:00–8:40	CT	Nikolay Abrosimov Sobolev Institute of Mathematics, Novosibirsk State University	The volume of a spherical antiprism with S_{2n} symmetry
14:40–14:50	8:40–8:50	Coffee break		
14:50–15:30	8:50–9:30	CT	Seongjeong Kim Jilin University	On parities for knots in $S_g \times S^1$
15:30–15:40	9:30–9:40	Coffee break		
15:40–16:20	9:40–10:20	CT	Sergey Melikhov Steklov Mathematical Institute	Topological isotopy and Cochran's derived invariants
16:20–16:50	10:20–10:50	Coffee break		
16:50–17:20	10:50–11:20	CT	Paolo Cavicchioli Università degli studi di Modena e Reggio Emilia	An algorithmic method to compute plat-like Markov moves for genus two 3-manifolds
17:20–17:30	11:20–11:30	Coffee break		
17:30–18:00	11:30–12:00	CT	Mohd Ibrahim Sheikh Pusan National University	Bikei's, Biquandles and Dichromatic Links

Tuesday, 30th November

Korea time	Moscow time			
14:00–14:40	8:00–8:40	PT	Sam Nelson Claremont McKenna College	Biquandle Bracket Quivers
14:40–14:50	8:40–8:50	Coffee break		
14:50–15:30	8:50–9:30	CT	Igor Nikonov Moscow State University	On the principle of indistinguishability for crossings of classical knots
15:30–15:40	9:30–9:40	Coffee break		
15:40–16:20	9:40–10:20	CT	Jieon Kim Pusan National University	On symmetric biquandles and invariants for unoriented surface-links
16:20–16:50	10:20–10:50	Coffee break		
16:50–17:20	10:50–11:20	CT	Bao Huu Vuong Tomsk State University	On hyperelliptic Euclidean 3-manifolds
17:20–17:30	11:20–11:30	Coffee break		
17:30–18:00	11:30–12:00	CT	Suhyeon Jung Pusan National University	Pseudoknots and the Goeritz matrix
18:00–18:10	12:00–12:10	Coffee break		

Wednesday, 1st December

Korea time	Moscow time			
14:00–14:40	8:00–8:40	PT	Louis Kauffman University of Illinois at Chicago	Invariants of Flat Virtual Knots and Links
14:40–14:50	8:40–8:50	Coffee break		
14:50–15:30	8:50–9:30	CT	Seung Yeop Yang Kyungpook National University	Set-theoretic Yang-Baxter cohomology groups of finite cyclic biquandles
15:30–15:40	9:30–9:40	Coffee break		
15:40–16:20	9:40–10:20	CT	Maxim Ivanov Novosibirsk State University	Recurrent construction of virtual link invariants arising from flat links
16:20–16:50	10:20–10:50	Coffee break		
16:50–17:20	10:50–11:20	CT		
17:20–17:30	11:20–11:30	Coffee break		
17:30–18:00	11:30–12:00	CT	Andrey Egorov Novosibirsk State University	On volumes of hyperbolic right-angled polyhedra
18:00–18:10	12:00–12:10	Coffee break		
18:10–18:50	12:10–12:50	PT	Vassily Olegovich Manturov Moscow Institute of Physics and Technology	Three faces of sliceness for knots: pictures, groups, matchings

Thursday, 2nd December

Korea time	Moscow time			
14:00–14:40	8:00–8:40	PT	Sergey Gukov California Institute of Technology	Non-semisimple TQFT's and BPS q-series
14:40–14:50	8:40–8:50	Coffee break		
14:50–15:30	8:50–9:30	PT	Sergei Lando Higher School of Economics, Skolkovo Institute of Science and Technology	Lie algebras weight systems and graph invariants
15:30–15:40	9:30–9:40	Coffee break		
15:40–16:20	9:40–10:20	CT	Seonmi Choi Kyungpook National university	On algebraic structures related to skein relations and their invariants
16:20–16:50	10:20–10:50	Coffee break		
16:50–17:20	10:50–11:20	CT	Tatyana Kozlovskaya Regional Mathematical Center, Tomsk State University	The singular pure braid group
17:20–17:30	11:20–11:30	Coffee break		
17:30–18:00	11:30–12:00	CT	Olga Frolkina M.V. Lomonosov Moscow State University	A solution of J.Cobb's question
18:00–22:00	12:00–16:00	Long break		
22:00–22:40	16:00–16:40	PT	Mikhail Khovanov Columbia University	Introduction to universal construction and foam evaluation

Friday, 3rd December

Korea time	Moscow time			
14:00–14:40	8:00–8:40	CT	Seonhwa Kim University of Seoul	Generalized Riley polynomials of a knot and trace fields of representations
14:40–14:50	8:40–8:50	Coffee break		
14:50–15:30	8:50–9:30	PT	Eiji Ogasa Meiji Gakuin University	Quantum Invariants of Links and 3-Manifolds with Boundary defined via Virtual Links
15:30–15:40	9:30–9:40	Coffee break		
15:40–16:20	9:40–10:20	CT	Vladimir Mikhajlovich Nezhinskij St Petersburg State University, Herzen University	Spatial framed graphs and their isotopy
16:20–16:50	10:20–10:50	Coffee break		
16:50–17:05	10:50–11:05	ST	Jin Seok Oh Kyungpook National university	Torsion subgroups of homology of finite dihedral quandles
17:05–17:20	11:05–11:20	ST	Hongdae Yun Kyungpook National University	The Betti numbers for the set-theoretic Yang-Baxter homology groups of Alexander biquandles

List of Abstracts – Talks

29th November

The volume of a spherical antiprism with S_{2n} symmetry.

Nikolay Abrosimov, Sobolev Institute of Mathematics and Novosibirsk State University

We consider a spherical antiprism. It is a convex polyhedron with $2n$ vertices in the spherical space S^3 . This polyhedron has a group of symmetries S_{2n} generated by a mirror-rotational symmetry of order $2n$, i.e. rotation to the angle $\frac{\pi}{n}$ followed by a reflection. We establish necessary and sufficient conditions for the existence of such polyhedron in S^3 . Then we find relations between its dihedral angles and edge lengths in the form of cosine rules through a property of a spherical isosceles trapezoid. Finally, we obtain an explicit integral formula for the volume of a spherical antiprism in terms of the edge lengths.

On parities for knots in $S_g \times S^1$.

Seongjeong Kim, Jilin University

For knots in $S_g \times S^1$, where S_g is an oriented surface of genus g , one of important information is “how many times a half of a crossing turns around S^1 , and we call it winding parity of a crossing. In this talk we extend this notion more generally and discuss its geometrical meaning.

Topological isotopy and Cochran’s derived invariants

Sergey Melikhov, Steklov Mathematical Institute

We construct a link in the 3-space that is not isotopic to any PL link (non-ambiently). Moreover, there exist uncountably many 1-equivalence classes of links in the 3-space. The proofs are based on Cochran’s derived invariants. We also note a formula expressing Cochran’s derived invariants in terms of the Conway polynomial (using bands). The details are available in arXiv:2011.01409.

An algorithmic method to compute plat-like Markov moves for genus two 3-manifolds

Paolo Cavicchioli, *Università degli studi di Modena e Reggio Emilia*

The talk deals with equivalence of links in 3-manifolds of Heegaard genus 2. Starting from a description of such a manifold introduced in [1], that uses 6-tuples of integers and determines a Heegaard decomposition of the manifold, we construct an algorithm (implemented in c++) which allows to find the words in $B_{2,2n}$, the braid group on $2n$ strands of a surface of genus 2, that realizes the plat-equivalence for links in that manifold. In this way we extend to the case of genus 2 the result obtained in [2] for genus 1 manifolds. We describe explicitly the words for a notable group of manifolds.

[1] Casali, M. R., Grasselli, L. 2-symmetric crystallizations and 2-fold branched coverings of S^3 . *Discrete Math.* 87, 9–22 (1991)

[2] Cattabriga, A., Gabrověk, B. A Markov theorem for generalized plat decomposition. *Ann.Sc. Norm. Super. Pisa Cl. Sci.* XX, 1273–1294 (2018)

Bikei's, Biquandles and Dichromatic Links

Mohd Ibrahim Sheikh, *Pusan National University*

Bikei and Biquandle are algebraic structures whose axioms are motivated by classical Reidemeister moves, and are used to construct invariants for classical knots and links. In this talk we will define their generalizations for dichromatic links and construct invariants for these links.

30th November

Biquandle Bracket Quivers

Sam Nelson, Claremont McKenna College

Biquandle brackets are skein invariants for biquandle-colored knots. Taken over the set of biquandle colorings of an oriented knot or link, the multiset of biquandle bracket values is an invariant from which can be recovered the biquandle counting invariant and, depending on the particular biquandle bracket, classical skein invariants as well as biquandle 2-cocycle invariants. A set of biquandle endomorphisms endows the multiset with an invariant quiver structure, proving a categorification of these invariants distinct from the Khovanov-style homological categorifications.

On the principle of indistinguishability for crossings of classical knots

Igor Nikonov, Moscow State University

A (weak chord) index is a function on the crossings of knot diagrams such that: 1) the index of a crossing does not change under Reidemeister moves; 2) crossings which can be paired by a second Reidemeister move have the same index. We show that one can omit the second condition in the case of the universal index. As a consequence, we get the following principle of indistinguishability for classical knots: crossings of the same sign in a classical knot diagram can not be distinguished by any inherent property.

On symmetric biquandles and invariants for unoriented surface-links

Jieon Kim, Pusan National University

A quandle is a non-empty set with a binary operation satisfying certain conditions derived from Reidemeister moves. A quandle can be generalized to a biquandle. When invariants are constructed by using quandles and biquandles, orientations are mostly needed. In that case, we only construct invariants for oriented (surface-)links. In 2009, Kamada and Oshiro constructed symmetric quandles, a quandle with a good involution. It is used to construct invariants for unoriented (surface-)links. In this talk, we introduce symmetric biquandles. By using this algebraic structure, we define invariants for unoriented (surface-)links.

On hyperelliptic Euclidean 3-manifolds

Bao Huu Vuong, Tomsk State University

We study closed orientable Euclidean manifolds which are also known as flat 3-dimensional manifolds or just Euclidean 3-forms. Up to homeomorphism, there are six of them. The first one is the three-dimensional torus. In 1972, R. H. Fox showed that the 3-torus is not a double branched covering of the 3-sphere. Then, it is not a hyperelliptic manifold. We show that all the remaining Euclidean 3-forms are hyperelliptic manifolds. This is a joint work with A. D. Mednykh.

Pseudoknots and the Goeritz matrix

Suhyeon Jeong, Pusan National University

In 1933, Goeritz described how a quadratic form could be obtained from a regular projection of a knot, and showed that some of the algebraic invariants of this form are invariants of the knot. In 2010, a pseudodiagram was introduced by Ryo Hanaki. A pseudodiagram is a knot or link diagram where we ignore over/under information at some crossings of the diagram. In 2012, Allison Henrich, Rebecca Hoberg, Slavik Jablan, Lee Johnson, Elizabeth Minten, and Ljiljana Radvic extended this idea of pseudodiagram to pseudoknots and pseudolinks, i.e. equivalence classes of pseudodiagrams modulo pseudo-Reidemeister moves. In this talk, we would like to introduce the Goeritz matrix for a checkerboard colored pseudodiagram of a pseudoknot or pseudolink, which is an extension of the Goeritz matrix for a checkerboard colored diagram of a knot or link. Using this, we show that the determinant, signature, and nullity of oriented classical knots and links extend to those of oriented pseudolinks. This is a joint work with Jieon Kim and Sang Youl Lee.

1st December

Invariants of Flat Virtual Knots and Links

Louis H Kauffman, University of Illinois at Chicago

This talk will discuss the structure of flat virtual knots and links. These correspond to immersions of curves in thickened surfaces taken up to 1-handle stabilization. The theory has a diagrammatic formulation with virtual crossings and flat crossings. Virtual crossings are allowed to make detour moves over all crossings, while flat crossings are restricted to detour only over flat crossings. We will discuss polynomial invariants of flat virtuals and their cobordism properties. We will also discuss what is presently known about reductions of flat virtual diagrams via the combinatorial moves.

Set-theoretic Yang-Baxter cohomology groups of finite cyclic biquandles

Seung Yeop Yang, Kyungpook National University

Set-theoretic Yang-Baxter (co)homology groups of biquandles and their cocycles can be applied to construct invariants of knots and links. In this talk, we determine the free parts completely and estimate the torsion parts of the integral set-theoretic Yang-Baxter cohomology groups of finite cyclic biquandles. This is joint work with Xiao Wang.

Recurrent construction of virtual link invariants arising from flat links

Maxim Ivanov, Novosibirsk State University

Theory of virtual knots and links was introduced by Kauffman as a generalization of classical knot theory. Flat virtual links are equivalence classes of virtual links with respect to a changing of a type of a classical crossing in a diagram. In 2018 Kaur, Prabhakar and Vesnin introduced a family of invariants of virtual knots, called F-polynomials, based on some invariants of flat virtual links. As a generalization of F-polynomials we present a recurrent construction of invariants of virtual link by using invariants of flat virtual links. Specifically we define weight functions which assign to every classical crossing in a diagram a value in an abelian group G , satisfying some natural conditions, analogous to Chord Index Axioms. A pair of those weights defines an invariant. We use a recursive procedure to construct various weights and hence, a sequence of invariants for virtual ordered links and flat ordered links. Those invariants appear to be useful in studying connected sums of virtual knots. As an example, we give a new proof of Kishino knot being nontrivial knot. This is a joint work with Andrei Vesnin, Amrendra Gill and Madeti Prabhakar (see arxiv:2111.04526).

On volumes of hyperbolic right-angled polyhedra

Andrey Egorov, Novosibirsk State University

In three-dimensional Lobachevsky space consider right-angled polyhedra. We will look at some properties of this polyhedra and consider new upper bounds on volumes of right-angled polyhedra in hyperbolic space in three different cases: for ideal polyhedra with all vertices on the ideal hyperbolic boundary, for compact polytopes with only finite vertices, and for finite volume polyhedra with vertices of both types. In addition, we will look at some connections with knot theory.

Three faces of sliceness for knots: pictures, groups, matchings

Vassily Olegovich Manturov, Moscow Institute of Physics and Technology

My talk will be devoted to sliceness obstructions for various analogues of knots. We shall be mostly concerned with the two cases:

1. Free knots (capped by formal folded 2-discs).
2. Knots in the full torus (to be capped by a disc in $S^1 \times D^3$).

A naive approach requires some "matchings" between crossings to be paired: say, for a 2-component link a mixed crossing can not be paired with a pure crossing. We'll see that one can strongly generalise the "matching" approach.

2nd December

Non-semisimple TQFT's and BPS q-series

Sergey Gukov, California Institute of Technology

Following recent work with Francesco Costantino and Pavel Putrov [arXiv:2107.14238], we will consider various perspectives on the operation of sending the quantum parameter q inside the unit disk to a root of unity. The goal is to see parallels between various manifestations of this limit in quantum groups, in vertex algebras, in exactly solvable lattice models, in the corresponding TQFTs, and in the geometry of Coulomb branches / affine Grassmannians.

Lie algebras weight systems and graph invariants

Sergei Lando, Higher School of Economics, Skolkovo Institute of Science and Technology

Knot invariants are functions on isotopy classes of knots. They are intended to distinguish knots. Vassiliev's theory of finite order knot invariants allows one to associate to each knot invariant a function on chord diagrams — simple combinatorial objects consisting of a circle and several chords in it. Such functions are called “weight systems”. Due to a theorem by Kontsevich, this correspondence is essentially one-to-one: each weight system determines a knot invariant. In particular, a weight system can be associated to any semi-simple Lie algebra. It happens, however, that already for the most simple nontrivial case, namely, for the Lie algebra \mathfrak{sl}_2 , the computations of the corresponding weight system are very complicated. This case is one of the most important ones because it corresponds to the famous knot invariant known under the name of colored Jones polynomial. The talk will be devoted to known results about computation of Lie algebra weight systems, including recent ones, as well as to open problems in the subject.

On algebraic structures related to skein relations and their invariants

Seonmi Choi, Kyungpook National University

Przytycki and Traczyk introduced a new algebraic structure, called the Conway algebra, and constructed invariants of oriented links valued in Conway algebras. Niebrzydowski and Przytycki defined a Kauffman bracket magma and constructed an invariant of framed links. These invariants are closely related to polynomial invariants. In this talk, we will define their generalizations for surface-links and construct invariants via marked graph diagrams. Moreover, we will define a specific map on a Kauffman bracket magma and construct some invariants for oriented links or oriented surface-links.

The singular pure braid group

Tatyana Anatolevna Kozlovskaya, Regional Mathematical Center, Tomsk State University

We suggest finite set of generators and defining relations for the singular pure braid group SP_n . Using this representation, we describe some properties of this group. Also we construct linear representations and representation by automorphisms of free group F_n for the singular braid group SB_n .

A solution of J.Cobb's question

Olga Frolkina, M.V. Lomonosov Moscow State University

Questions about the projections of zero-dimensional sets were considered already at the end of the 19th century. In 1884 G.Cantor described the surjection of the middle-thirds Cantor set onto the segment $[0, 1]$; its graph is zero-dimensional, but gives the unit segment when projecting on the Oy-axis. Cantor sets in plane all of whose projections are segments were constructed by L.Antoine (1924), H.Otto (1933), A.Flores (1933), G.Noebeling (1933). In 1947, K.Borsuk described a Cantor set in \mathbb{R}^N , whose projection onto any hyperplane contains an $(N-1)$ -dimensional ball, equivalently, has dimension $(N - 1)$. In 1994, J.Cobb constructed a Cantor set in \mathbb{R}^3 , whose projection onto any 2-plane is one-dimensional, and posed a general question: does there exist, for given numbers $N > m > k > 0$, a Cantor set in \mathbb{R}^N , whose projection onto any m -plane is of dimension k ? (Briefly: (N, m, k) -set.) For the cases $(N, m, m - 1)$ and $(N, N - 1, k)$ such sets were constructed by O.Frolkina (2010) and S.Barov, J.J.Dijkstra, M.van der Meer (2012), respectively. Different examples for particular cases $(N, N - 1, N - 1)$ and $(N, N - 1, N - 2)$ were given by O.Frolkina (2020-21).

J.Cobb also asked: Cantor sets that raise dimension under all projections and those in general position with respect to all projections are both dense in the Cantor sets in \mathbb{R}^N - which (if either) is more common, in the sense of category or dimension or anything?

We answer the category part of this question, showing that a generic Cantor set in \mathbb{R}^N has general position with respect to all projections. As a corollary, we obtain our earlier result: all projections of a typical Cantor set are Cantor sets.

Introduction to universal construction and foam evaluation

Mikhail Khovanov, Columbia University

Foams are two-dimensional cobordisms in 3-space between planar graphs that naturally appear in constructions of link homology theories. We will review foams and their evaluations, that are used in a combinatorial approach to link homology. Foam evaluation utilizes universal construction of topological theories, that give rise to lax TQFTs, where the state space of the union of manifolds properly contains tensor products of their state spaces.

3rd December

Generalized Riley polynomials of a knot and trace fields of representations

Seonhwa Kim , University of Seoul

We generalize R. Riley's study about parabolic representations of two bridge knots to the general knots in S^3 . A generalized Riley polynomial $R_c(y) \in \mathbb{Q}[y]$ is defined for any knot diagram with a specified base crossing c , where the roots also correspond to conjugacy classes of parabolic representations as like the original Riley's. In particular, as the *sign-type* of parabolic quandle is newly introduced, we obtain a formula for the obstruction class to lift a boundary unipotent $SL(2, C)$ -representation and moreover, we can define another polynomial $g_c(u) \in \mathbb{Q}[u]$, called *u-polynomial*, and prove that $R_c(u^2) = \pm g_c(u)g_c(-u)$. Based on this investigation, we introduce *Riley field* and *u-field* closely related to the invariant trace field of representations.. Finally we will consider several open questions related to them.

Quantum Invariants of Links and 3-Manifolds with Boundary defined via Virtual Links

Eiji Ogasa, Meiji Gakuin University

We use virtual links, and introduce new topological quantum invariants of compact oriented 3-manifolds with boundary where the boundary is a disjoint union of two identical surfaces. By using our new invariants, we give a new invariant of classical knots and links in the 3-sphere. Virtual link invariants make new classical knot and link invariants. These invariants are new, nontrivial, and calculable.

Spatial framed graphs and their isotopy

Vladimir Mikhailovich Nezhinskij, St Petersburg State University, Herzen University

The problem of isotopy classification of spatial framed graphs equipped with an additional structure - a skeleton, an oriented vertex and a marked point on its boundary, is reduced to the problem of isotopy classification of tangles. The results are contained in [1] and [2].

[1]. V. M. Nezhinskij, Isotopy invariants of spatial graphs, Siberian Electronic Mathematical Reports, 17 (2020), 769 - 776.

[2]. V. M. Nezhinskij, Spatial graphs and their isotopy classification, Siberian Electronic Mathematical Reports (manuscript submitted for publication)

Torsion subgroups of homology of finite dihedral quandles

Jinseok Oh, Kyungpook National University

The free parts of rack and quandle homology were completely determined by Litherland and Nelson, Etingof and Graña independently. However, little is known that the torsion parts are related to the orders of torsion elements in rack homology. For example, it is known that the order of a finite quasigroup quandle annihilates the torsion subgroup of its rack homology. In this talk, we will discuss the torsion subgroups of homology of dihedral quandles. This is joint work with Seung Yeop Yang.

The Betti numbers for the set-theoretic Yang-Baxter homology groups of Alexander biquandles

Hongdae Yun, Kyungpook National University

The Yang-Baxter equation was introduced independently by C. N. Yang(1967) and R. J. Baxter(1972), and it has become important role in the study of knot theory and quantum physics, etc. Biquandles are special families of solutions to the set-theoretic Yang- Baxter equation. A homology theory for the set-theoretic Yang-Baxter equation was introduced by Carter, Elhamdadi, and Saito. In this talk, we first review the definition of set- theoretic Yang-Baxter (co)homology and determine the Betti numbers for the set-theoretic Yang-Baxter (co)homology groups of some finite Alexander biquandles. This is joint work with Seung Yeop Yang, Jinseok Oh, Donghan Kim.

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